DEPRECIATION RESERVES AS AFFECTED BY PROPERTY GROWTH

It is now almost universally recognized among producing industries that some regular provision should be made for replacement of equipment worn out in operation or superseded by more efficient or suitable types in the course of industrial and social development. In spite of this general recognition of depreciation the methods of providing for it are widely dissimilar. While in some cases renewals are not provided for until made, the prudent manager of a prosperous enterprise systematically builds up reserves far in advance of actual replacement requirements.

It is the purpose of this paper to show that certain methods of accumulating such reserves in common use, while economically sound under some conditions, fail to recognize a factor of varying but sometimes very great importance. The result of neglect of this factor may be that conscientious executives will accumulate excessive reserves, thereby depriving stockholders of the full returns to which they are entitled. While errors, if any, may have been more common in the opposite direction, giving larger returns to stockholders than really belonged to them, such errors have usually resulted from failure to analyze carefully the depreciation problem rather than from oversight of any of the important factors which properly enter into such analyses. In this discussion of the subject application will be limited largely to public utilities because their consistent and rapid growth makes them particularly good illustrations. The same reasoning is equally applicable to other industries which have similar characteristics.

Because of the comparatively short life and rapid development of the utility industry much of the equipment employed therein up to this time has had very limited usefulness. It is probable, however, that in the future there will be a closer agreement between natural life, determined by ordinary wear and decay, and useful life, which is fixed by all elements contributing toward ultimate replacement. It should therefore be possible in the future to make more definite provision for replacements from income with rates that are at the same time reasonable and sufficiently high to take care of this feature sometimes neglected in the past.

This past neglect has not all been ignorance or carelessness of operating officials or undue greed of investors. There were visions, now partially realized, of a vast development of business which

would permit liberal future provisions for depreciation with rates for service which were not exorbitant. If full provision had been required in the early years of the business when actual replacement expenditures required by ordinary wear and decay were small, rates for service would have been so high as to retard seriously or even prohibit the development of much needed public service.

The earliest definite step taken by the utilities in providing for depreciation, as in many other fields of activity, was the reservation of a portion of the annual net profit before dividends were declared. Such reservations were not definitely fixed but were increased or diminished with good or bad business conditions. If in a term of years the total reservation was sufficient, there remained the definite advantage of a comparatively stable return on the investment. As time went on, some utilities transferred their depreciation allowances from surplus to a distinct depreciation reserve definitely held for replacement requirements.

The procedure above described is open to the objection that it is indefinite and may not in the long run be adequate. There is always a temptation to distribute too liberal dividends, hoping for better business conditions in the future to build up the depreciation reserve. This objection may be met by maintaining a definite average during a term of years sufficiently long to include the usual cycle of good and bad business. This involves the necessity of determining what constitutes a suitable average annual depreciation reserve.

Some utilities which have carefully studied the subject have adopted a production basis as a standard, setting aside each year a certain amount per kilowatt hour, per car mile, per thousand feet of gas, etc. Other utilities have attempted to estimate the useful life of their property and have set aside each year a corresponding percentage of its value. Still others, not having had, perhaps, suitable inventories of their properties but recognizing a fairly definite relation between property value and gross earnings, have set aside each year a percentage of their gross for depreciation. This last method has the advantage of partial adjustment of the depreciation reserve to the prosperity of the business.

With the advent of regulation there has developed a demand on the part of the commissions for more definite and scientific bases for insuring adequate provisions for depreciation. This movement has culminated in a requirement of the Interstate Commerce Commission that all carriers under its jurisdiction shall set aside

every month during the life of certain prescribed elements of physical property or groups of similar elements hereafter acquired, a sum representing the total average reduction in value during that period. If an element has an estimated life of twenty years, each month must see 1/240 of its value written off. Most of the state commissions, which have otherwise generally followed the accounting methods prescribed by the Interstate Commerce Commission, have not definitely adopted this method of depreciation accounting. They doubtless recognize that it can not be fully applied to previously acquired property and that it results in the accumulation of a large reserve that is never needed.

Until a property reaches a period when its elements have acquired uniformly distributed ages, the annual expenditures for replacements will not rise to the normal uniform point. Thereafter, they will average an approximately constant amount equal to the annual reserve set aside therefor. In the earlier years there will be an increasing total accumulation, rapid at first, but gradually diminishing in rate as short-life elements are replaced. The unused accumulation may approach or even exceed 50 per cent of the value of the property. This fund is contributed by the patrons of the utility and, without any restriction on its use, as is the case in interstate accounting, the free money represented thereby is usually invested in additions to the property. There is no logical reason why patrons should be required to furnish funds for physical additions to utility property without assurance of a definite return thereon. Their only return is through reduction in rates, so that without both close regulation and continued regular patronage their return will be less than might be obtained from an investment of their own choosing.

Without commission requirement few utilities would adopt the practice above described but would rather set aside, without interest accumulations, a percentage of gross or of investment which increasing experience shows would take care of replacements as and when necessary. Percentages of this kind as commonly used take into consideration the character and life expectancy of the particular property involved. It is not necessary to start a depreciation reserve during the earliest years of operation, as an accumulation started after say ten years, when the business has become comparatively stable, will be found sufficient for normal requirements. Most utilities did not start such reserves from their beginning and would now find any other basis than present value difficult or impossible of full application.

In so far as depreciation percentages are based upon the full value of the property at the present time (or gross earnings bearing a fairly definite relation thereto) they neglect a factor of very considerable importance, namely, the rate of growth of the property. Replacements are made today of elements placed in service say 20 years ago. Perhaps those elements comprised 1/20 of the property then in use, or 5 per cent. If the property has been growing, their value will be by no means 5 per cent of the value today. A property valued at $1,000,000 today, if of steady growth at the rate of 6 per cent per year, was worth 20 years ago only $312,000; and replacements amounting to 5 per cent of that value are less than 2 per cent of the value today. It is therefore readily seen that the rate of growth is a factor to be carefully considered in any provisions for depreciation based directly or indirectly on the present-day value of the property.

It is now proposed to analyze the effect of the rate of growth as carefully as possible, assuming certain average conditions, and to draw certain conclusions therefrom applicable to actual properties. The results are embodied in formulas which express the annual cost of replacements of worn out or obsolete property in terms of (1) the value of the property as a whole at the time of the inquiry, (2) the useful life of its depreciable elements, (3) the range of life if uniformity is lacking, in other words, the difference in useful years between the shortest and longest-lived elements, and (4) the rate at which the property has been increasing in value during the life of its existing elements. After these formulas have been developed the effect upon the annual cost of replacements of varying any of the factors involved is readily determined. Variations in the last-named factor, the rate of growth, constitute, as already stated, the particular field of the present investigation.

The analysis employed is a purely mathematical one and the various average and uniform conditions necessarily assumed are not actually found in any existing public utility. The fundamental assumptions with respect to the property are as follows:

1. A uniform rate of growth.
2. A uniformly distributed age of all depreciable elements.
3. A uniform useful life of all depreciable elements, or a range of life which is uniformly distributed with respect to both number and value of such elements.
4. An unlimited age of the property as a whole.

No one of these assumptions is fully met in any property. The

last is particularly lacking in applicability to public utilities comparatively few of which, with the exception of gas and water works, are over 30 years old. The extent of the error in connection with this last assumption will, however, be shown. Departures from uniformity in the other assumptions can not be definitely measured but their effect can to some extent be estimated. In spite of this, no claim is made for other than broad and general applicability of the method employed.

The analysis involves the tracing of each element or group of similar elements replaced in any year back to its date of installation and expressing the cost of its replacement as a percentage of the total present value of the property. The sum of such percentages for different elements or groups of different elements replaced during the year gives the total percentage of present value spent for replacements.

The method may be first illustrated by the very simple example of a property without growth, with all elements of uniform life and uniformly distributed age. Each year will see the replacement of one group of elements, the useful life of which has ended. If the useful life is 20 years, the cost of replacements each year will be 1/20, or 5 per cent of the total value of the property.

If there is a range of life also uniformly distributed, of say 15 years, each year will see the replacement of 15 groups of elements, installed between 14 and 28 years previous. Each one of these groups represents 1/300 of the total value of the property and as there are 15 groups the total annual cost of replacements remains 1/20, or 5 per cent of the total property value.

Turning now to the more complicated case of a growing property, to which subsequent consideration is limited, a somewhat different treatment is found to be necessary. With an assumed uniform rate of growth from a time of zero value, and uniform life and uniformly distributed age of all elements, the replacements in any year may be divided into groups including, respectively, the elements which were new n years ago (n representing the years of useful life) and replaced for the first time, the elements which were new $2n$ years ago and replaced for the second time, the elements which were new $3n$ years ago and replaced for the third time, and so on until the value of the elements becomes negligible because of the constantly decreasing size of the property as it is traced farther and farther back. All replacements made in any year are included in some one of these groups.

With uniform rate of growth or increase in property value (which will be represented by i, a decimal) the relation between present value (V) and the value n years ago (v_1) may be expressed by the formula:

$$V = v_1 (1 + i)^n$$

The value of v_1 in this formula is

$$v_1 = \frac{V}{(1+i)^n} \text{ or } V(1+i)^{-n}$$

The value $2n$ years ago is similarly found to be:

$$v_2 = \frac{v_1}{(1+i)^n} = \frac{V}{(1+i)^{2n}}.$$

Values of v_3, v_4, v_5, etc., follow the same form, for example:

$$v_5 = \frac{V}{(1+i)^{5n}}.$$

Values of $(1 + i)^n$ and $\frac{1}{(1+i)^n}$ may be found in published compound interest tables for ordinary ranges of both i and n.

The annual additions to property are i times the present value. If in any year all elements first installed n years ago are to be replaced, their value is $i \times v_1$; or $\frac{iV}{(1+i)^n}$. If in addition there are to be replaced a second time elements installed $2n$ years ago, their value is similarly $i \times v_2$ or $\frac{iV}{(1+i)^{2n}}$. The third replacement of elements installed $3n$ years ago will cost $\frac{iV}{(1+i)^{3n}}$.

In a property of indefinite age with uniform life of all elements the total cost of replacements in a year in which the total value of the property is V will be the sum of the series of such members carried to the point of negligible value, or

$$C = iV \left(\frac{1}{(1+i)^n} + \frac{1}{(1+i)^{2n}} + \frac{1}{(1+i)^{3n}} + ---- 0 \right).$$

The percentage of the value V spent in replacements, which it is desired to determine, is

$$x = \frac{C}{V} = i \left\{ \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{2n}} + \frac{1}{(1+i)^{3n}} + ---- 0 \right\}.$$

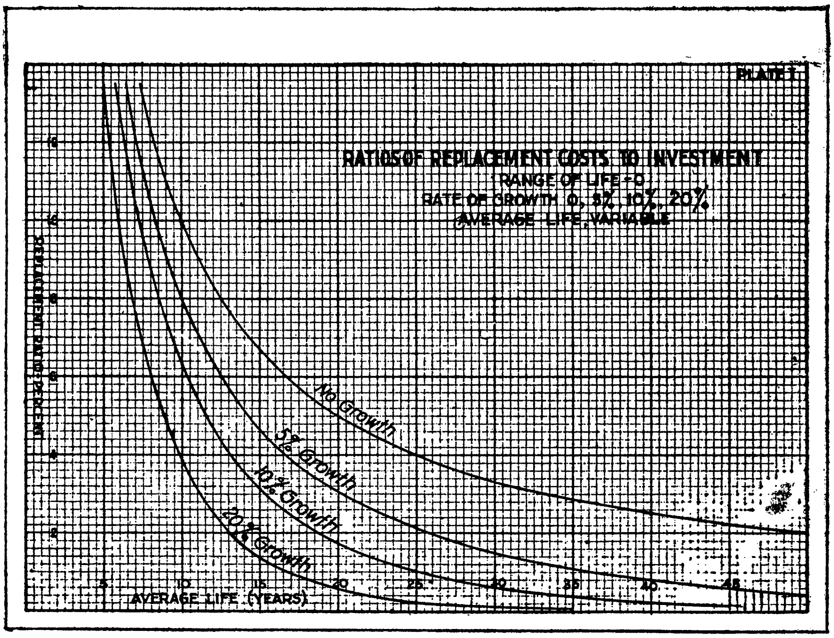
This formula applies to any rate of growth above zero, at which point it assumes an indefinite form, $0 \times \infty$. As already shown, when $i = 0$, $C = \frac{V}{n}$ and $x = \frac{C}{V} = \frac{V}{nV} = \frac{1}{n}$.

As an example of the application of this formula the ratio of replacement cost to present value will be determined for a property

growing in value steadily at the rate of 5 per cent per year with a uniform life of all elements of 25 years. The value of $\frac{1}{(1+i)^n}$ when $i = 0.05$ and $n = 25$ is found from the tables to be 0.295, $\frac{1}{(1+i)^{2n}}$, determined from tables or calculated is 0.087, $\frac{1}{(1+i)^{3n}}$ is 0.025, etc. The sum of the complete series is 0.419. The ratio, x, is therefore found as follows:

$$x = i \times 0.419 = 0.05 \times 0.419 = 0.0209, \text{ or } 2.09 \text{ per cent.}$$

If n is varied through the range of life usually encountered in public service property, x may be determined for various values and corresponding points plotted on a curve representing a fixed 5 per cent value of i. Other values of i may then be selected and corresponding curves prepared. Such curves for several values of i, covering the range usually encountered in public service property, are shown in accompanying Plate I.



The curve of no growth shows values of x equal always to $\frac{1}{n}$. Thus for a 25-year life, $x = 4$ per cent; for 16 2/3 years, $x = 6$ per cent, etc. With a 10 per cent rate of growth, however, with a 25-year life, x is only 1 per cent, instead of 4 per cent with no

growth. The enormous effect of the rate of growth is at once apparent and the inconsistency of depreciation provisions depending only upon present value or revenue becomes clear.

Where there is both increase in value of property from year to year, and variation in useful life of the elements, the almost universal condition in public service properties, a definite expression of the annual cost of replacements is more complicated. It is most readily determined by assuming that the range of life is uniformly distributed over a certain number of years. If the range is r years, there will be r series of factors, similar in form to those described in the preceding section, in each of which the useful life will be different, ranging from the years of the shortest-lived group of elements to those of the longest-lived. The value of each group of elements is $\frac{1}{r}$ of the value of the groups previously considered where the useful life was uniform. But, as stated, there are r times as many groups entering into the total cost of replacements.

The complete formula expressing the percentage of present value annually spent for replacements, with uniform increase in value and uniformly distributed range of life of elements within assumed limits is as follows, n representing the average useful life.

$$x = \frac{i}{r}\left\{\begin{array}{l}
\frac{1}{(1+i)^{n-\frac{r-1}{2}}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n-\frac{r-1}{2}+1}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}+1\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}+1\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n-\frac{r-1}{2}+2}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}+2\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}+2\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n-\frac{r-1}{2}+3}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}+3\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}+3\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n-\frac{r-1}{2}+4}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}+4\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}+4\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n-\frac{r-1}{2}+5}} + \frac{1}{(1+i)^{2\left(n-\frac{r-1}{2}+5\right)}} + \frac{1}{(1+i)^{3\left(n-\frac{r-1}{2}+5\right)}} + \cdots 0 \\
+ \text{ other series in a total of } r \text{ series, the last two being as follows:—} \\
+ \frac{1}{(1+i)^{n+\frac{r-1}{2}-1}} + \frac{1}{(1+i)^{2\left(n+\frac{r-1}{2}-1\right)}} + \frac{1}{(1+i)^{3\left(n+\frac{r-1}{2}-1\right)}} + \cdots 0 \\
+ \frac{1}{(1+i)^{n+\frac{r-1}{2}}} + \frac{1}{(1+i)^{2\left(n+\frac{r-1}{2}\right)}} + \frac{1}{(1+i)^{3\left(n+\frac{r-1}{2}\right)}} + \cdots 0
\end{array}\right\}$$

The calculation of all the factors in the above formula would be quite laborious as there may be several thousand under some conditions. Fortunately each of the r series is of the form $a + a^2 + a^3 + \text{----} 0$ which, when a is less than unity as in this case, is equal to $\frac{a}{1-a}$. The first member of each series, here represented by a, is taken directly from compound interest tables. The whole equation may therefore be reduced to the following simpler form:

$$x = \frac{i}{r}\left\{ \frac{\frac{1}{(1+i)^{n-\frac{r-1}{2}}}}{1-\frac{1}{(1+i)^{n-\frac{r-1}{2}}}} + \frac{\frac{1}{(1+i)^{n-\frac{r-1}{2}+1}}}{1-\frac{1}{(1+i)^{n-\frac{r-1}{2}+1}}} + \frac{\frac{1}{(1+i)^{n-\frac{r-1}{2}+2}}}{1-\frac{1}{(1+i)^{n-\frac{r-1}{2}+2}}} + \cdots \frac{\frac{1}{(1+i)^{n+\frac{r-1}{2}}}}{1-\frac{1}{(1+i)^{n+\frac{r-1}{2}}}} \right\}$$

The commonly accessible compound interest tables, prepared for financial and engineering purposes, are not sufficiently comprehensive for extended depreciation calculations. Tables prepared for life insurance calculations should not be used as they are based on *advance* premium payments. For convenient use in connection with this subject complete new tables have been calculated to fully cover the range to be expected in depreciation problems.

Table A, appended, gives values of $\frac{1}{(1+i)^n}$ for useful values of i between 1 per cent and 15 per cent, and for similar values of n between 1 and 100 years. Although some intermediate values of both i and n are omitted, complete calculations are possible for the ordinary ranges encountered in public service property. For other or longer-lived kinds of property approximate calculations over a wider range are also possible. $\frac{1}{(1+i)^n}$ is simply the amount which, at the assumed rate of growth, will accumulate to unity in any number of years under consideration.

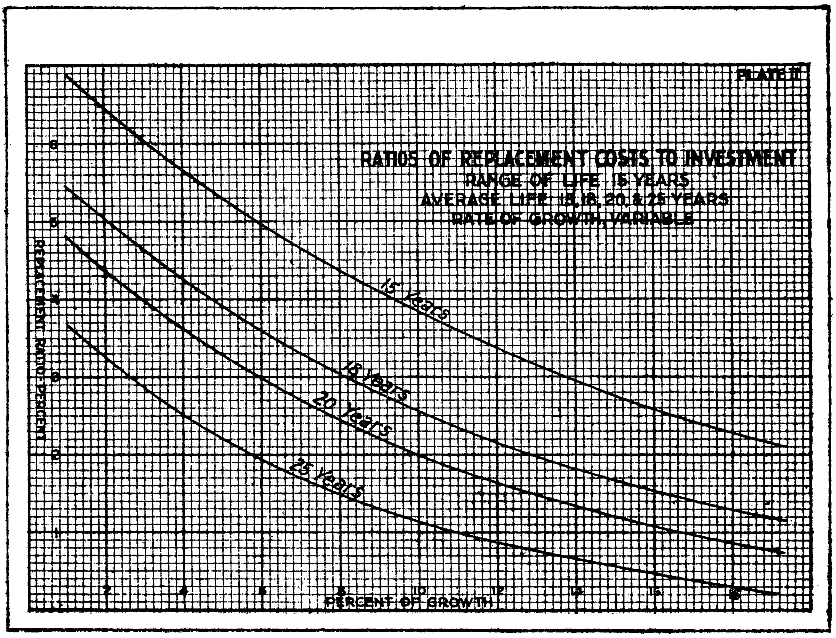
In calculations involving assumed unlimited age it is not a value of $\frac{1}{(1+i)^n}$ that is needed but the sum of an infinite series of which $\frac{1}{(1+i)^n}$ is the first member, the form of this series being, as already stated, $a + a^2 + a^3 + - - - - 0$. Table B has, therefore, been calculated for the same ranges as Table A, showing the sums of the infinite series of which the values of $\frac{1}{(1+i)^n}$ in Table A are the first members. With this table the calculation of depreciation ratios becomes very simple. As both these tables were calculated mechanically, exact accuracy is not assured, but possible errors are less than would arise from other approximations and assumptions necessary in working out any practical problem to which the tables are applicable.

The actual working out of an example will assist in making the procedure clear. The example will be to find the percentage of present value spent annually for replacements in the case of a property growing at the rate of 6 per cent per year, with an average life of elements of 20 years and a range of life of these elements of 15 years. With the range assumed the shortest-lived elements

are replaced after 13 years and the longest-lived after 27 years. The value of $\frac{1}{(1+i)^{n-\frac{r-1}{2}}}$ when $i = 0.06$, $n = 20$ and $r = 15$, becomes $\frac{1}{(1+i)^{13}}$ and is found in Table A to be 0.469. The sum of the series of which this is the first member is $\frac{0.469}{1 - 0.469}$, which from Table B is found to be 0.883. In the final series, in which $\frac{1}{(1+i)^{n+\frac{r-1}{2}}} = \frac{1}{(1+i)^{27}}$, the first member is found to be 0.208 and the sum of the series 0.262. Without tracing the determination of the 13 intermediate series the sum of all 15 series may be found by adding directly from the 6 per cent column in Table B (years 13 to 27 inclusive) to be 7.455. From this sum the value of x is determined as follows:

$$x = \frac{i}{n} \times 7.455 = \frac{.06}{15}\, 7.455 = .0298, \text{ or } 2.98 \text{ per cent.}$$

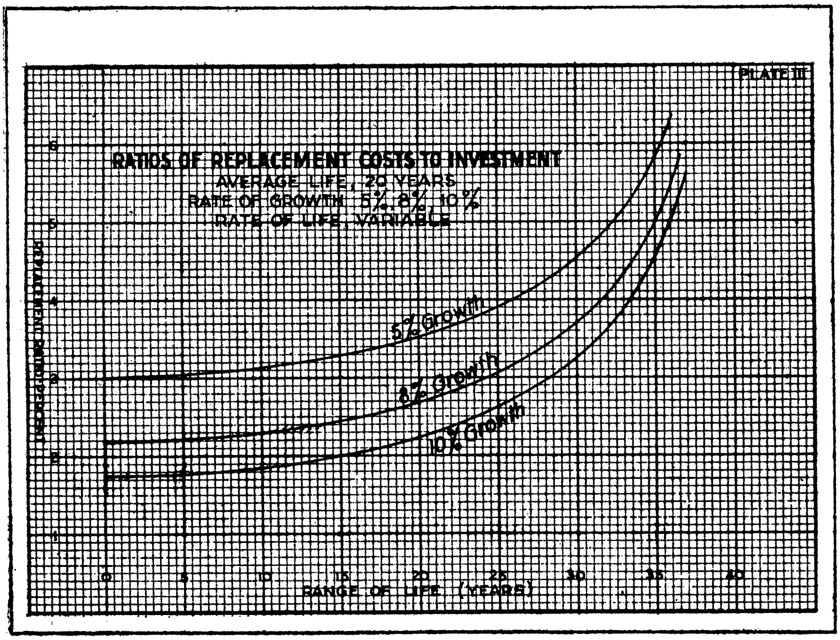
If the rate of growth, i, in the above example is changed, other factors remaining the same, a series of values of x may be obtained and plotted in a curve. Changing also the values of n, other curves representing various average useful lives may be obtained. Several of these are shown in Plate II. An examination of these



curves shows again the very marked effect of growth of a property upon the relation between replacement costs and total property value. With a 20-year average life and a 10 per cent annual growth, the ratio is only 2 per cent instead of about 5 per cent with no growth. A ratio of 1 per cent is found for a rate of growth of 16 2/3 per cent per year.

It will be noticed on Plate II that the curves are not plotted through to points of zero growth. If extended they would not show exactly the ratio values which might be expected. In the case of the 20-year life and no growth above referred to the ratio would not be exactly 5 per cent. This is due to the 15-year range of life which is assumed in all these curves.

It will be of interest to study the effect of changes in range of life upon the replacement ratio. This may be done by changing the values of r in the general formula, other factors being kept constant. Plate III shows the results with several different rates



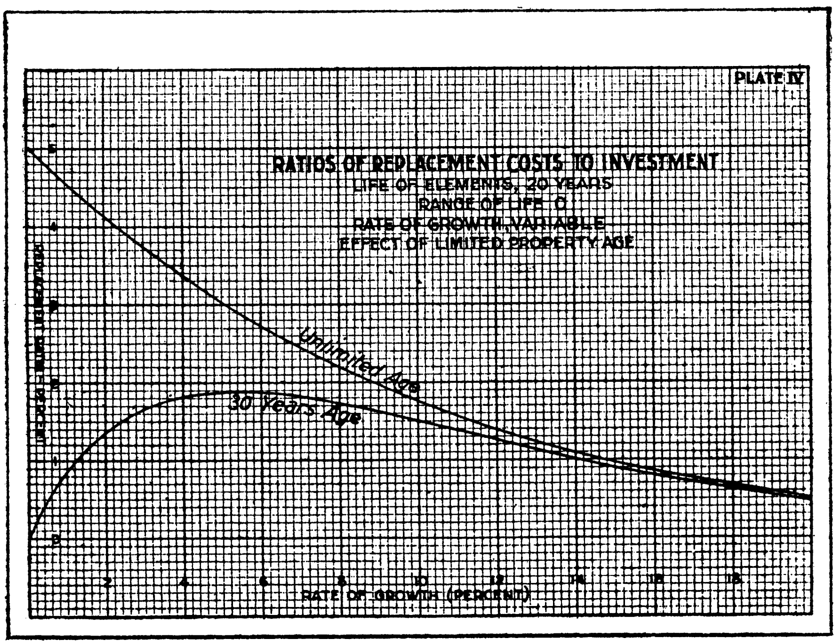
Rate of Life in title should read Range of Life.

of growth, the average life in all cases being the same. It appears that within moderate limits variations in the life of different elements, which do not affect the average life of all, change the replacement ratio comparatively little, but when there is a very

wide range, involving very frequent replacement of certain elements, the replacement ratio increases quite rapidly. With an average life of 20 years and a range of life of 35 years certain elements have a life of only 3 years ($20 - \frac{35-1}{2}$) and replacement of such elements at this short interval is necessary. By the use of Plate III curves the useful range of Table II may be considerably extended.

It was stated at the beginning of this discussion that some of the assumptions necessary to a mathematical analysis were not met in actual properties and that with respect to age they were not approximated. An unlimited age was assumed, whereas actual life has been comparatively short. It is possible to show, approximately at least, the extent of the error involved when the age of the property is limited as suggested to 30 years. This is done by cutting off in the general replacement ratio formula all members in each series which involve new elements or replaced elements which had their origin more than thirty years back. The condensed form of the general formula cannot be used in this study as individual members in each series are required.

Plate IV shows a general ratio curve of a property having a uni-



form life of all elements of 20 years, and an unlimited age. There is plotted below this the curve of a property similar except as to its age, which is limited to 30 years. It appears that properties which have grown faster than 10 per cent per year show a small departure from the theoretical or unlimited replacement ratio. At 8 per cent growth the discrepancy is slightly more than 20 per cent and with less growth it increases rapidly. In all cases the actual ratio is less than the theoretical. The zero percentage of replacement at zero growth shown by the 30-year age curve, while theoretically true for uniform 20-year life, would not actually be found in any practical cases. If a property less than 30 years old is considered, the discrepancy from the condition of unlimited age is increased to the extent of the increase in number of elements which have not reached the limit of usefulness within the assumed period.

It has been stated herein that departure from uniformity in other factors than that referred to above could not be definitely measured. While this is true with respect to irregular variations in general, the effect of certain departures can be measured. Referring to the general formula it is seen that the uniform rate of growth (i) appears uniformly in all members and can not be varied. The same is true of average life (n) and range of life (r). However, the assumption is made that the values represented in all series are the same; in other words, that the short-life elements have values equal to the long-life elements. It is probable that in the long run there is greater and more consistent departure in practice from this assumption than from those of uniform growth and constant average life.

In the case of a property growing at the rate of 10 per cent, with an average life of elements of 20 years, it is found from Plate II that the replacement ratio for a uniformly distributed range of life of 15 years is 2 per cent. If instead of assigning equal values to each of the 15 different life groups, represented by the 15 series in the formula, different values are selected, the replacement ratio can still be calculated although with some increased labor.

A sample calculation has been made assuming that the shorter life groups of elements represent less than $\frac{1}{15}$ of the total value and the longer life groups correspondingly more than $\frac{1}{15}$. This is the condition commonly found in public service property, partly because of the greater permanence naturally attached to the more expensive units and partly because the renewals of short-life units

are charged to operating expense and do not burden the depreciation reserve. The latter is not, however, a valid consideration in a strictly theoretical analysis.

Assuming the value of the longest life group of elements as 50 per cent greater than the average and the shortest life group 50 per cent less and arranging a uniform variation between these extremes, the total value not being changed, the replacement ratio is found to be 1.71 per cent, or about 14 per cent less than with uniformly distributed values. It is not necessary in these calculations to assume any uniform relation between the values in the different groups, and the complication of calculation is not materially increased by any desired arbitrary distribution of total value. For such use of the general formula its form should be changed to the following, in which negative exponents have also been employed for further general simplification:

$$x = i\left\{ p_1 \frac{(1+i)^{-\left(n-\frac{r-1}{2}\right)}}{1-(1+i)^{-\left(n-\frac{r-1}{2}\right)}} + p_2 \frac{(1+i)^{-\left(n-\frac{r-1}{2}+1\right)}}{1-(1+i)^{-\left(n-\frac{r-1}{2}+1\right)}} + p_3 \frac{(1+i)^{-\left(n-\frac{r-1}{2}+2\right)}}{1-(1+i)^{-\left(n-\frac{r-1}{2}+2\right)}} + \cdots p_r \frac{(1+i)^{-\left(n+\frac{r-1}{2}\right)}}{1-(1+i)^{-\left(n+\frac{r-1}{2}\right)}} \right\}$$

In the above form p_1, p_2, p_3 - - - - p_r are the percentages of total value of the property belonging in the r different life groups. Calculations based upon actual distribution of value, if this is determinable in any case, would generally show ratios, as in the example given, less than the theoretical ratio. The general statement may therefore be made that a depreciation reserve set up in accordance with the unmodified general formula will liberally provide for all ordinary replacement requirements.

One factor touched upon herein needs further consideration. There are three kinds of expenditures made upon physical property to maintain indefinitely its usefulness: (1) those which merely insure safe and efficient current operation; (2) those which have a tendency to prolong useful life; (3) those required to replace by equivalent units those elements of the property which have outlived their usefulness. In common accounting practice the first two classes of expenditures are charged to operating expense although the second tends to reduce replacement costs which are included in the third class. The third class includes all ex-

penditures for entire replacement of units including those for which advance provision in the form of a depreciation reserve is necessary. Not all expenditures of this class are ordinarily handled through a depreciation reserve. It is common practice to charge renewals of minor, short-life elements direct to operating expense. A railway company will handle ties, poles, trolley wire and perhaps rail, special work and other miscellaneous items in this way. Other classes of utilities have similar but less extensive charges. All such companies have real estate which is not depreciable, also working capital, supplies, etc., included in their appraisals, and possibly some overhead charges which are not recurrent in connection with renewals.

An analysis of typical appraisals of utility property shows that the elements which do not require consideration in depreciation accounting amount to about 25 per cent of the total value of an average composite property. Street railways alone have a higher percentage, electric light and gas companies lower percentages.

It therefore appears that in applying the replacement ratios herein determined to property values to find the sum to be set aside annually for depreciation, the value V should be 25 per cent or some other specifically determined percentage less than the total investment. In one of the illustrations used herein a replacement ratio of 2.0 per cent was found. If applied to the whole investment, instead of to only the depreciable portion not accounted for in operating expense, this ratio is reduced to about 1.5 per cent. If applied for any reason to capitalization it might be still further reduced.

It will be of interest in conclusion to compare the procedure determined by the foregoing analysis with the actual handling of depreciation requirements in the case of a particular public utility. This company, in common with many of its kind, has been in the habit of reserving each year before declaring dividends a portion of its profit (after providing for operating expenses, taxes, and interest charges) amounting to approximately 10 per cent of its gross earnings, less bond sinking fund requirements. The company has added to its physical property at the rate of 6 per cent per year. The appraised value of the entire property is only four times the annual gross, the business being well developed.

If the average useful life of the depreciable property is 20 years and the range of this life is 15 years, we find from Plate II that the corresponding replacement ratio is very close to 3 per cent. If 25

per cent of the property included in the inventory requires no replacement or is taken care of in operating expenses, there remains 75 per cent of the 3 per cent, or $2\frac{1}{4}$ per cent of the appraised value to be used annually for replacements or set aside in a depreciation reserve. If the total appraised value is four times the gross, the percentage of gross which should be reserved is $4 \times 2\frac{1}{4}$ per cent, or 9 per cent. It appears, therefore, that the 10 per cent habitually reserved is 11 per cent in excess of actual average requirements.

A somewhat less favorable case may now be considered, namely, a property with the same physical characteristics as above but growing only at the rate of 2 per cent per year and having gross earnings only 1/5 the value of its physical property. The full replacement ratio is found from Plate II to be 4.36 per cent. If this is an electric light and power property the percentage of value excluded from the depreciation provisions may be only 20 per cent, leaving 80 per cent of 4.36 per cent, or 3.49 per cent as the corrected replacement ratio. With total investment five times the gross, the ratio of replacement cost to gross is 5×3.49 per cent, or 17.5 per cent. If this company followed the practice of setting aside 10 per cent of its gross before paying dividends, its reserve would in the long run be entirely inadequate. If it has so far escaped embarrassment it may be attributed to its youth or to the fact that it still retains, perhaps with questionable economy, much of its original equipment theoretically beyond the useful age.

The effect of salvage from abandoned elements has so far not been mentioned. It may amount to 10 per cent or more of the original cost of the elements or may in other cases be negligible. By assuming that V as used herein is the "wearing value" instead of total cost any desired allowance for salvage may be made.

One more problem of a somewhat different form will be outlined, involving the determination of the proper ratio of annual replacement reserve to property value and gross earnings. An assumed street railway property now has gross earnings of $650,000 per year. Ten years ago it was earning only $250,000. Its rapid development has necessitated the early abandonment of substantial parts of its equipment so that the average life of the whole is not over 18 years. The range of life is probably not far from 15 years.

The mean rate of growth during the 10 years is determined from Table A by finding the per cent column to which the growth

factor lies nearest in the ten-year line. The growth factor is $\frac{250,000}{650,000} = 0.385$ and is almost exactly at the 10 year, 10 per cent point in Table A. Assuming a 10 per cent rate of growth, the replacement ratio is found on the 18-year curve, Plate II, to be 2.56 per cent. With 30 per cent of the total value not provided for in the depreciation reserve the net ratio becomes 1.79 per cent. This calculation assumes that the property value has increased at the same rate as the gross earnings. This is not usually the case on account of possible increase in saturation of the investment as business develops. In this assumed case, however, an unusual amount of unproductive expenditures for paving and other civic improvements has tended to keep the ratio nearly constant. A constant ratio of investment to gross of 4.75 is assumed, making a ratio of replacement cost to earnings of $1.79 \times 4.75 = 8.5$ per cent. So long as the assumed conditions obtain, this company should set aside not less than 8½ per cent of its annual gross, less any sinking or improvement fund provisions, for present or future replacements of property. Without the rapid growth which this company has enjoyed the ratios obtained would be materially higher and would involve a serious burden because of the comparatively low gross yield from the investment.

It is not within the scope of this paper to discuss the physical and engineering problems arising in depreciation studies. One reference to this phase of the subject can not, however, be omitted without leaving a possibly magnified impression of the effect of growth upon depreciation reserves. It has been shown that rapid increase in property value from year to year permits a very large reduction in per cent of total value required for replacement, other things being equal. But some of these other things do not remain equal. Rapid growth usually involves outgrown and otherwise inadequate equipment and consequent shorter average life than is found with more conservative development. A shorter average life means a higher replacement ratio. Therefore its effect is to counteract to some extent that resulting from rapid growth. Under ordinary conditions the influence of growth should largely predominate so that it can not properly be neglected.

While there is every reason to expect continued substantial growth in all fields of utility activity, we have now reached a stage in development which will permit more systematic provision for depreciation. It is now universally recognized that utility patrons and not investors should carry this burden, and with increasingly

developed business they can do so without hardship. If as a basis of determining what this burden shall be, utilities are permitted and choose to use total investment, or present earnings, kilowatt hours, car miles, etc., derived therefrom, the foregoing discussion may be of assistance in determining suitable percentages or factors. It is not applicable to cases in which reserves are started for each element or group of similar elements at the time of installation, based on their actual cost or their depreciable value.

It should at least be clear that a particular utility desiring to establish a suitable depreciation reserve should not adopt a certain percentage of its gross, or a certain amount per car mile operated or per kilowatt hour generated because some other utility rendering similar service has found by experience that these figures produce adequate provision for replacements as required. Not only may the properties have different characteristics affecting useful life, but also the communities served may be so diverse in progressiveness and expansion that radically different expenditures may be required by the utilities to keep pace with the general development. Such factors should therefore be carefully considered in determining depreciation reserves and the practice of other utilities should not be adopted without assurance of similarity in physical property and development history. Each case should preferably be considered in the light of its own particular characteristics, with due recognition of the factors discussed herein.

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Table A.—*Compound Interest Table.*

Amounts which, compounded annually, will accumulate to unity in various years at certain rates.

Years	1%	2%	3%	4%	5%	6%	7%	8%	10%	15%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.909	0.870
2	.980	.961	.943	.925	.907	.890	.874	.857	.826	.756
3	.971	.942	.915	.889	.864	.840	.816	.794	.751	.658
4	.961	.924	.888	.855	.823	.792	.763	.735	.683	.572
5	.951	.906	.863	.822	.784	.747	.713	.681	.621	.497
6	.942	.888	.837	.790	.746	.705	.666	.630	.565	.432
7	.933	.871	.813	.760	.711	.665	.623	.584	.513	.376
8	.923	.854	.789	.731	.677	.628	.582	.540	.467	.327
9	.914	.837	.766	.703	.645	.592	.544	.500	.424	.284
10	.905	.820	.744	.676	.614	.559	.508	.463	.386	.247
11	.896	.804	.722	.650	.585	.527	.475	.429	.351	.215
12	.887	.789	.701	.625	.557	.497	.444	.397	.319	.187
13	.878	.773	.681	.601	.531	.469	.415	.368	.290	.163
14	.870	.758	.661	.578	.505	.443	.388	.341	.263	.141
15	.861	.743	.642	.555	.481	.418	.363	.315	.239	.123
16	.853	.728	.623	.534	.458	.394	.339	.292	.218	.107
17	.844	.714	.605	.513	.436	.372	.317	.270	.198	.093
18	.836	.700	.587	.494	.416	.351	.296	.250	.180	.081
19	.828	.686	.570	.475	.396	.331	.277	.232	.164	.070
20	.819	.673	.554	.456	.377	.312	.258	.215	.149	.061
21	.811	.660	.538	.439	.359	.294	.242	.199	.135	.053
22	.803	.647	.522	.422	.342	.278	.226	.184	.123	.046
23	.795	.634	.507	.406	.326	.262	.211	.170	.112	.040
24	.787	.622	.492	.390	.310	.247	.197	.158	.102	.035
25	.780	.610	.478	375	.295	.233	.184	.146	.092	.030
26	.772	.598	.464	.361	.281	.220	.172	.135	.084	.026
27	.764	.586	.450	.347	.268	.208	.161	.125	.076	.023
28	.757	.574	.437	.334	.255	.196	.150	.116	.069	.020
29	.749	.563	.424	.321	.243	.185	.141	.107	.063	.017
30	.742	.552	.412	.308	.231	.174	.131	.099	.057	.015
31	.734	.541	.400	.297	.220	.164	.123	.092	.052	.013
32	.727	.531	.388	.285	.210	.155	.115	.085	.048	.011
33	.720	.520	.377	.274	.200	.146	.107	.079	.043	.010
34	.713	.510	.366	.264	.190	.138	.100	.073	.039	.009
35	.706	.500	.355	.254	.181	.130	.094	.068	.036	.008
36	.699	.490	.345	.244	.173	.123	.088	.063	.033	.007
37	.692	.481	.335	.234	.164	.116	.082	.058	.030	.006
38	.685	.471	.325	.225	.157	.109	.076	.054	.027	.005
39	.678	.462	.316	.217	.149	.103	.071	.050	.024	.004
40	.672	.453	.307	.208	.142	.097	.067	.046	.022	.004
42	.658	.435	.289	.193	.129	.087	.058	.040	.018	.003
44	.645	.418	.272	.178	.117	.077	.051	.034	.015	.002
45	.639	.410	.264	.171	.111	.073	.048	.031	.014	.002
46	.633	.402	.257	.165	.106	.069	.044	.029	.013	.002
48	.620	.387	.242	.152	.096	.061	.039	.025	.010	.001
50	.608	.372	.228	.141	.087	.054	.034	.021	.009	.001
60	.550	.305	.170	.095	.054	.030	.017	.010	.003	. .
70	.498	.250	.126	.064	.032	.017	.009	.005	.001	. .
75	.474	.227	.109	.053	.025	.013	.006	.003	.001	. .
80	.451	.205	.094	.043	.020	.009	.004	.002	.001	. .
90	.408	.169	.070	.029	.012	.005	.002	.001	. .	. .
100	.370	.138	.052	.020	.008	.003	.001	. .	. .	. .

TABLE B.—*Sums of Infinite Series.*

This table contains the sums of the infinite series, the first members of which are the figures in Table A for the corresponding years and percentages.

Yrs.	1%	2%	3%	4%	5%	6%	7%	8%	10%	15%
1	100.000	50.000	33.333	25.000	20.000	16.667	14.286	12.500	10.000	6.667
2	49.761	24.773	16.422	12.263	9.753	8.091	6.905	6.007	4.760	3.100
3	33.014	16.360	10.779	8.001	6.342	5.234	4.447	3.850	3.021	1.920
4	24.704	12.140	7.960	5.890	4.640	3.810	3.219	2.773	2.155	1.335
5	19.618	9.616	6.273	4.615	3.619	2.957	2.486	2.131	1.638	0.988
6	16.272	7.921	5.150	3.769	2.940	2.390	1.998	1.704	1.296	.762
7	13.881	6.727	4.350	3.165	2.457	1.986	1.651	1.401	1.054	.602
8	12.038	5.825	3.746	2.713	2.095	1.685	1.393	1.175	0.874	.486
9	10.655	5.127	3.280	2.362	1.814	1.451	1.193	1.001	.736	.397
10	9.537	4.568	2.906	2.083	1.591	1.265	1.034	0.863	.628	.329
11	8.625	4.110	2.601	1.854	1.408	1.114	0.905	.751	.540	.274
12	7.865	3.728	2.348	1.664	1.257	0.988	.799	.659	.468	.230
13	7.224	3.405	2.134	1.504	1.130	.883	.710	.582	.408	.194
14	6.674	3.129	1.951	1.367	1.021	.794	.634	.516	.358	.165
15	6.199	2.888	1.792	1.249	0.927	.717	.569	.460	.315	.140
16	5.784	2.681	1.653	1.145	.846	.650	.513	.412	.278	.120
17	5.419	2.499	1.532	1.055	.774	.591	.464	.371	.247	.103
18	5.090	2.336	1.424	0.975	.711	.540	.420	.334	.219	.088
19	4.797	2.189	1.327	.904	.655	.494	.382	.302	.196	.076
20	4.534	2.058	1.240	.840	.605	.454	.348	.273	.175	.065
21	4.296	1.939	1.163	.782	.560	.417	.318	.248	.156	.056
22	4.081	1.830	1.092	.730	.519	.384	.292	.226	.140	.048
23	3.882	1.734	1.027	.683	.483	.355	.267	.205	.126	.042
24	3.702	1.643	0.968	.640	.450	.328	.246	.187	.113	.036
25	3.535	1.561	.914	.600	.419	.304	.226	.171	.102	.031
26	3.382	1.485	.864	.564	.391	.282	.208	.156	.092	.027
27	3.241	1.415	.819	.531	.366	.262	.192	.143	.083	.024
28	3.108	1.350	.777	.500	.342	.244	.177	.131	.075	.020
29	2.986	1.289	.737	.472	.321	.227	.163	.120	.067	.017
30	2.872	1.233	.701	.446	.301	.211	.151	.110	.061	.015
31	2.766	1.180	.667	.421	.283	.179	.140	.101	.055	.013
32	2.664	1.130	.635	.399	.266	.183	.130	.094	.050	.011
33	2.570	1.084	.605	.378	.250	.171	.120	.086	.045	.010
34	2.482	1.041	.577	.358	.235	.160	.111	.079	.041	.009
35	2.400	1.000	.551	.340	.221	.150	.103	.073	.037	.008
36	2.319	0.962	.527	.322	.209	.140	.096	.067	.034	.007
37	2.245	.925	.504	.306	.197	.131	.089	.062	.030	.006
38	2.175	.891	.482	.291	.186	.123	.083	.057	.027	.005
39	2.108	.859	.461	.277	.175	.115	.077	.052	.025	.004
40	2.044	.828	.442	.263	.166	.108	.072	.048	.023	.004
42	1.927	.771	.406	.239	.148	.095	.062	.041	.019	.003
44	1.818	.719	.374	.217	.132	.083	.054	.035	.015	.002
45	1.770	.695	.360	.207	.125	.078	.050	.032	.014	.002
46	1.722	.673	.345	.197	.118	.074	.046	.030	.013	.002
48	1.632	.630	.319	.180	.106	.065	.040	.026	.011	.001
50	1.550	.591	.296	.164	.095	.057	.035	.022	.009	.001
60	1.224	.439	.204	.105	.057	.031	.018	.010	.003	. .
70	0.993	.334	.145	.069	.033	.017	.009	.005	.001	. .
75	.900	.293	.122	.056	.026	.013	.006	.003	.001	. .
80	.822	.258	.104	.045	.020	.009	.004	.002	.001	. .
90	.690	.203	.075	.030	.012	.005	.002	.001	. .	. .
100	.586	.160	.055	.020	.008	.003	.001	. .	. .	. .